

Probability and Random Processes

ECS 315

Asst. Prof. Dr. Prapun Suksompong

prapun@siit.tu.ac.th

3 Classical Probability



Office Hours:

BKD 3601-7

Monday 14:00-16:00

Wednesday 14:40-16:00

Real coins are biased

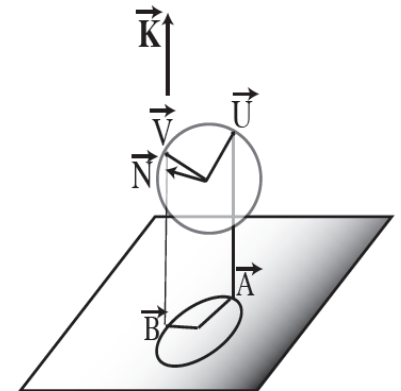
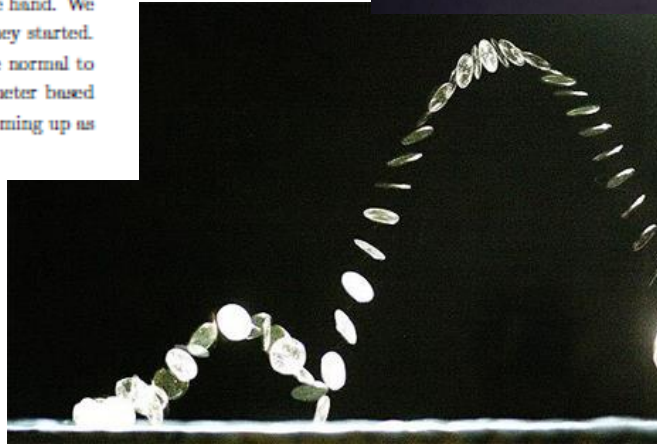
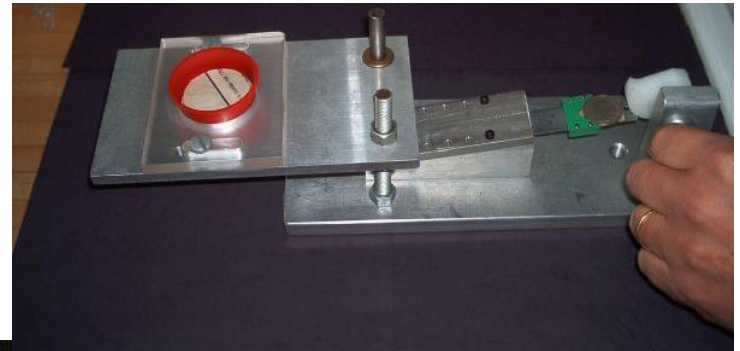
- From a group of Stanford researchers

DYNAMICAL BIAS IN THE COIN TOSS

Persi Diaconis	Susan Holmes	Richard Montgomery
Departments of Mathematics and Statistics	Department of Statistics	Department of Mathematics
Stanford University	Sequoia Hall	University of California
	Stanford University	Santa Cruz

Abstract

We analyze the natural process of flipping a coin which is caught in the hand. We prove that vigorously-flipped coins are biased to come up the same way they started. The amount of bias depends on a single parameter, the angle between the normal to the coin and the angular momentum vector. Measurements of this parameter based on high-speed photography are reported. For natural flips, the chance of coming up as started is about .51.

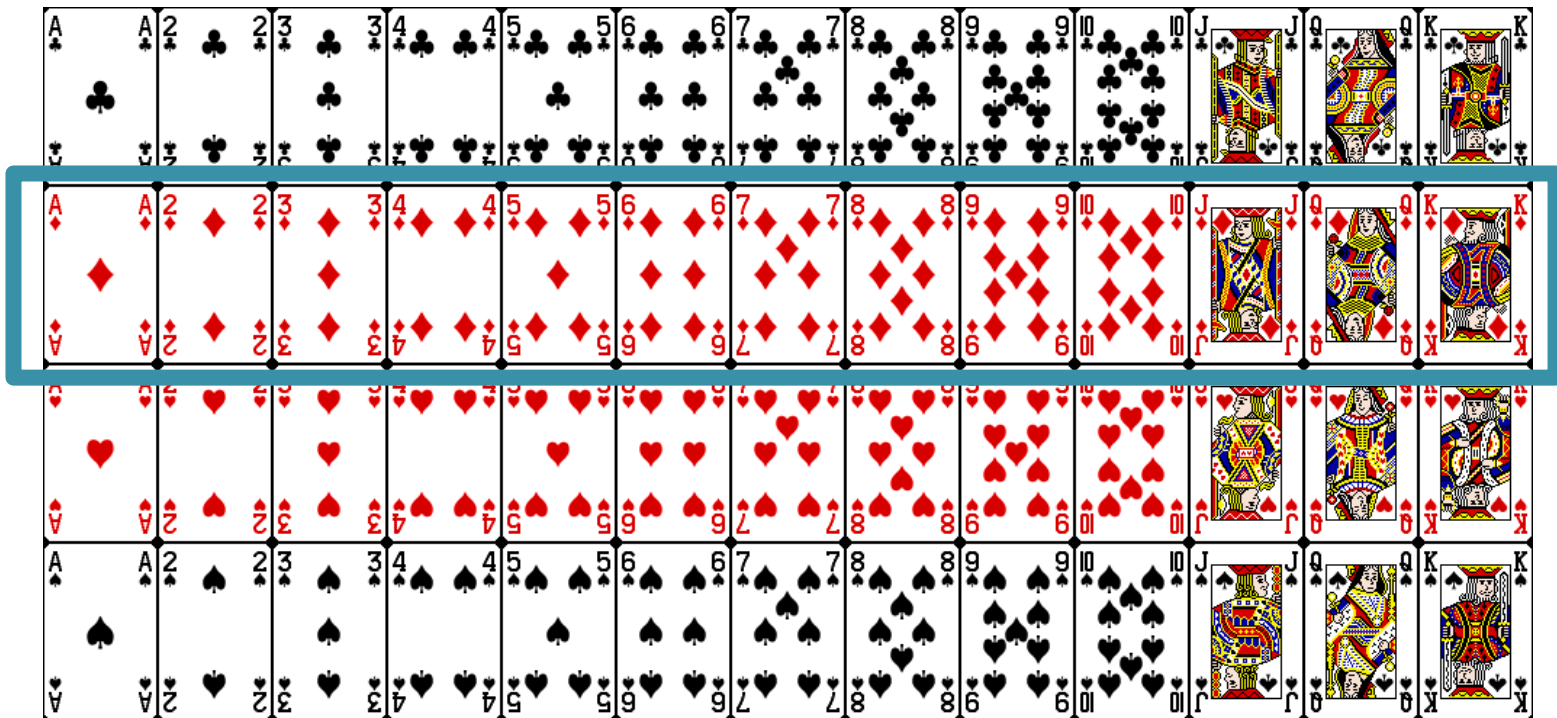


<http://gajitz.com/up-in-the-air-coin-tosses-not-as-neutral-as-you-think/>
<http://www.codingthewheel.com/archives/the-coin-flip-a-fundamentally-unfair-proposition>
<http://www-stat.stanford.edu/~susan/papers/headswithJ.pdf>



Example

- In drawing a card from a deck, there are 52 equally likely outcomes, 13 of which are **diamonds**. This leads to a probability of $13/52$ or $1/4$.



The word “dice”

- Historically, **dice** is the plural of **die**.
- In modern standard English, **dice** is used as both the singular and the plural.



Example of 19th Century bone dice



“Advanced” dice

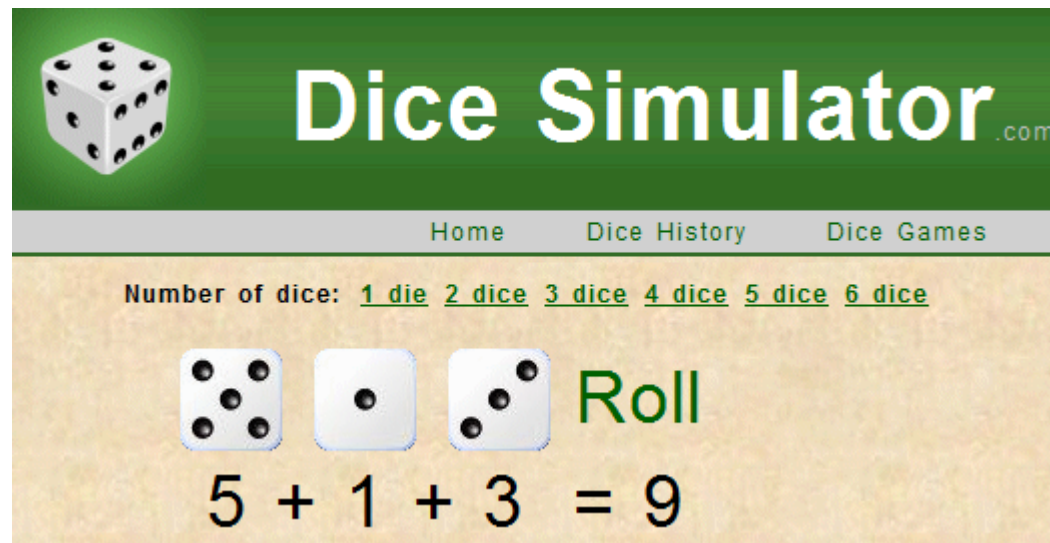


[<http://gmdice.com/>]



Dice Simulator

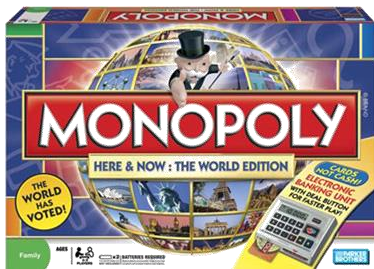
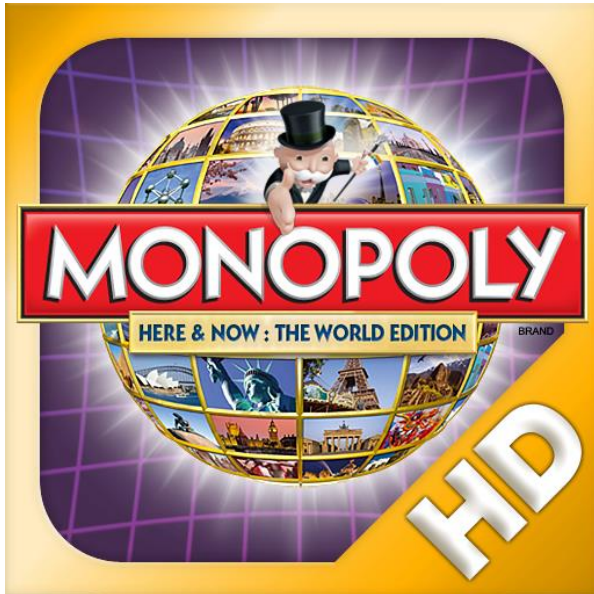
- <http://www.dicesimulator.com/>
- Support up to 6 dice and also has some background information on dice and random numbers.



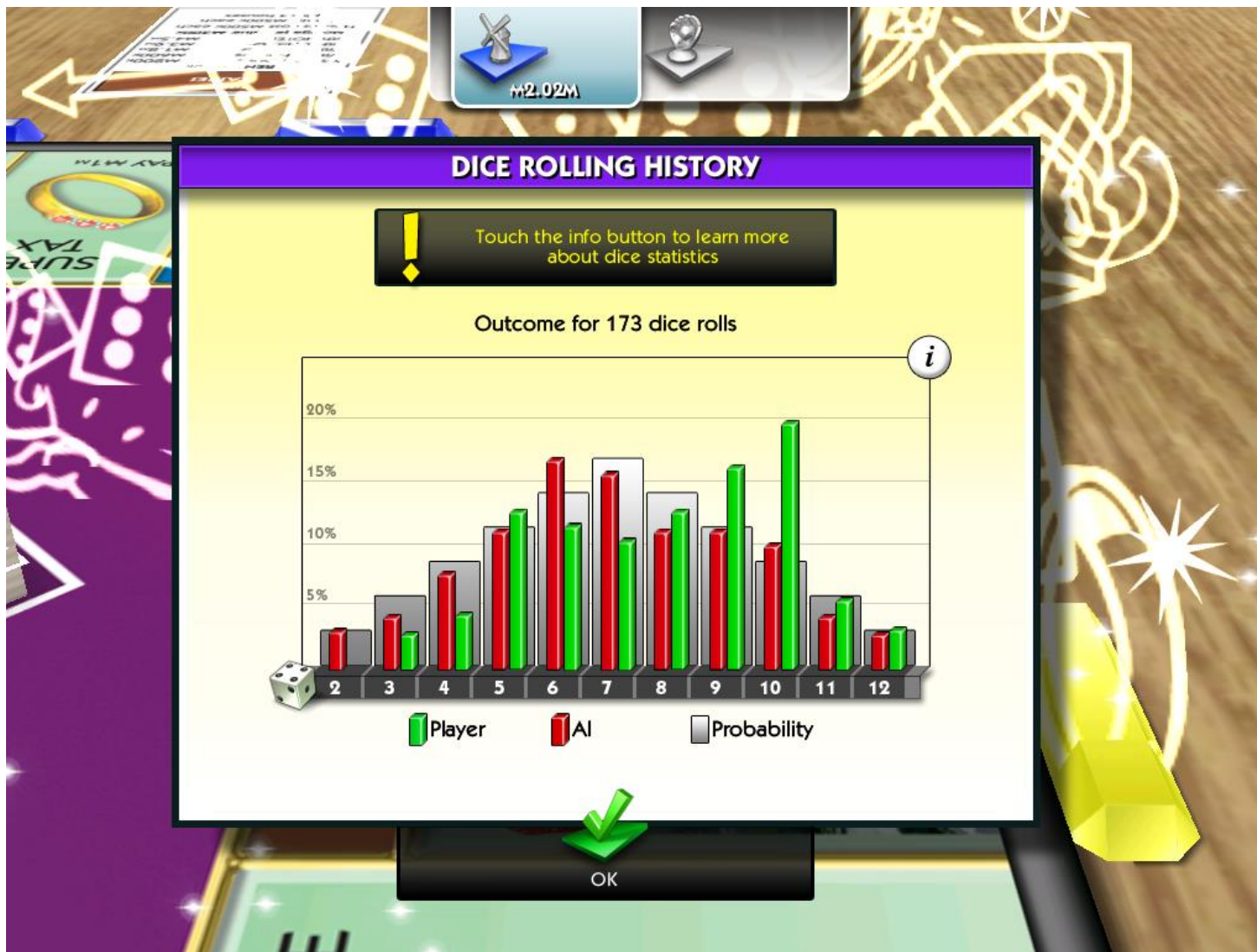
The screenshot shows the website's interface. At the top left is a 3D die icon. The main title "Dice Simulator" is in large white font on a green background, with ".com" in smaller text to the right. Below the title is a navigation bar with "Home", "Dice History", and "Dice Games" links. Underneath is a section for "Number of dice:" with links for "1 die", "2 dice", "3 dice", "4 dice", "5 dice", and "6 dice". The main content area shows three dice faces with 5, 1, and 3 dots respectively, followed by the word "Roll" in green. Below this is the equation $5 + 1 + 3 = 9$.



Two Dice

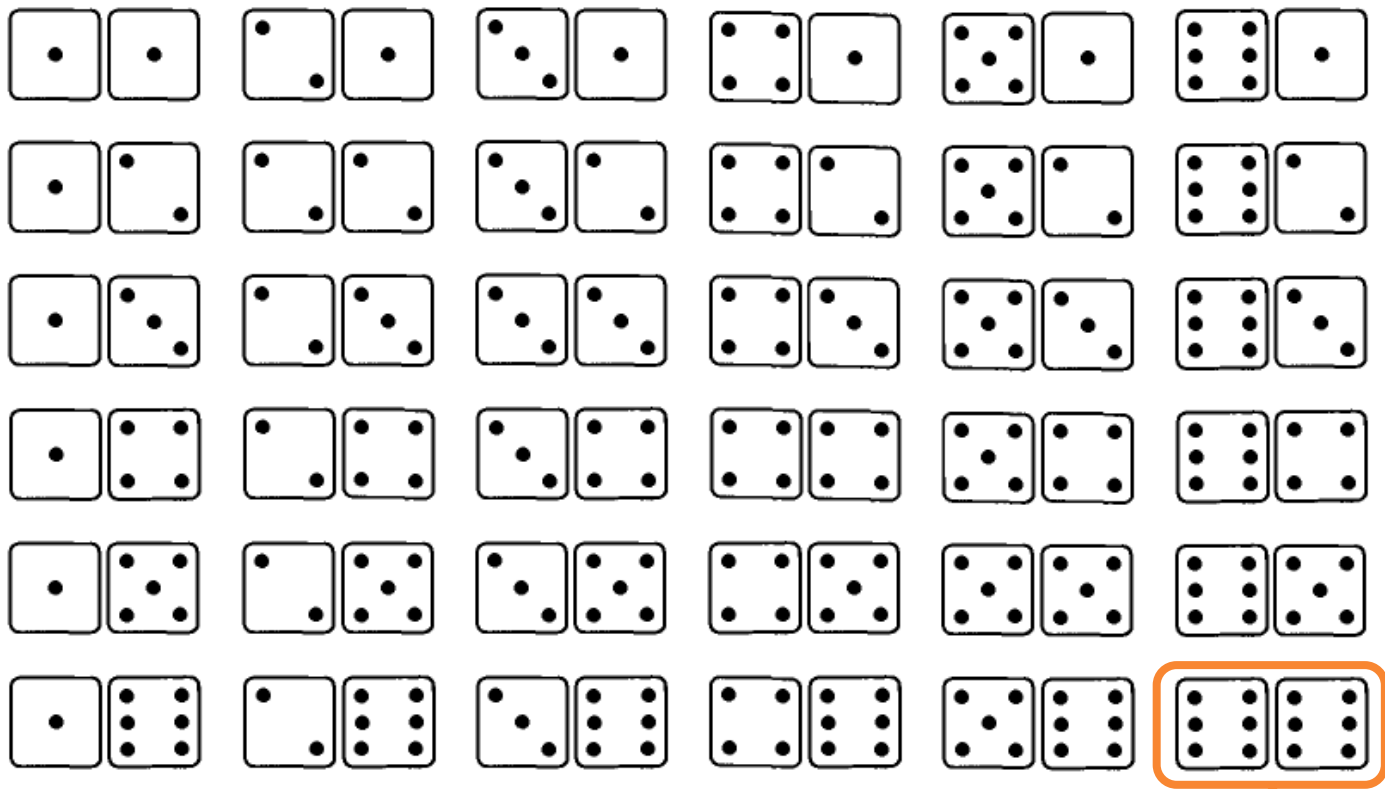


Two-Dice Statistics



Two Dice

- A pair of dice




Double six

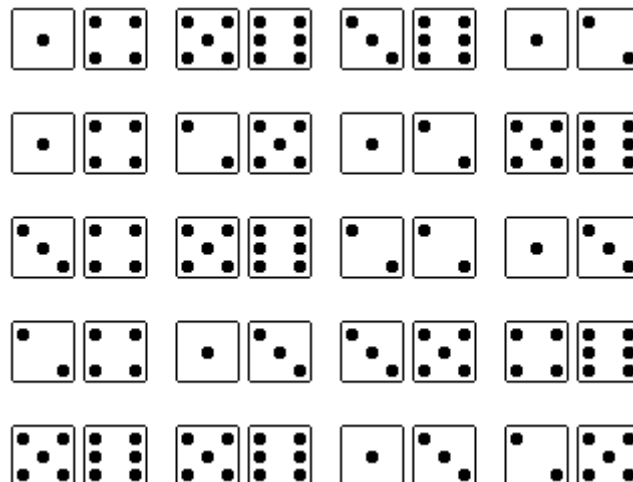


Two dice: Simulation



 <i>Simulated Experimental Dice-Roll Data (2 dice)</i>
Roll how many sets of 2 Dice? <input type="text" value="20"/> <input type="button" value="Roll Them!"/>
The results of the dice rolls will appear in a pop-up window. If you have pop-ups disabled, you might have to check to see if another window opened in the background.
<input type="button" value="Reset Form"/>
©Jeff LeMieux, 2002

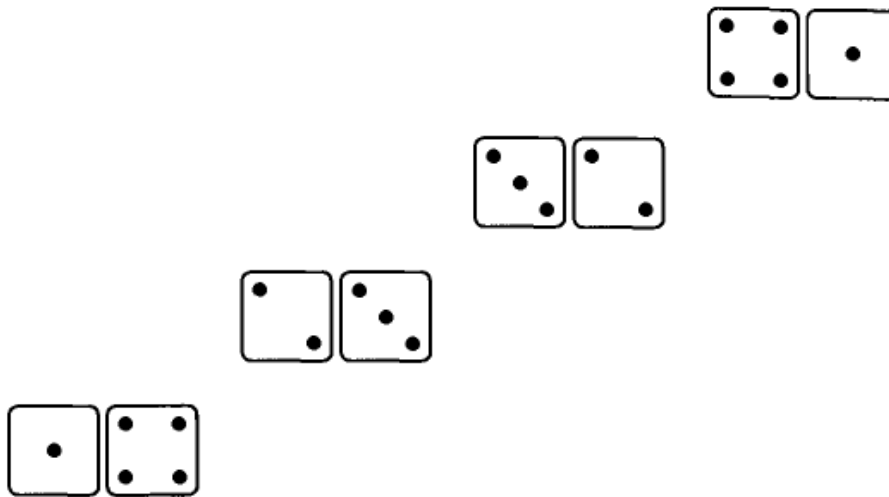
[<http://www2.whidbey.net/ohmsmath/webwork/javascript/dice2rol.htm>]



Two dice

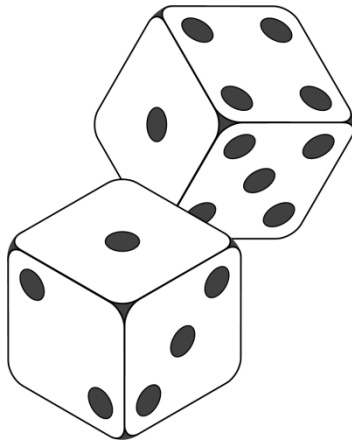


- Assume that the two dice are fair and independent.
- $P[\text{sum of the two dice} = 5] = 4/36$



Two dice

- Assume that the two dice are fair and independent.



DICE CHART		
ROLL		PROBABILITY ↗
2		1/36
3		2/36
4		3/36
5		4/36
6		5/36
7		6/36
8		5/36
9		4/36
10		3/36
11		2/36
12		1/36



Two-Dice Statistics

